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**Exact Solution of Bianchi Type- $V$  Model with Variable  
Cosmological Term- $\Lambda$  and  $G$**



A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in  
Physics

By

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# Dedication

The dedication of this work goes to the beloved persons whom that I won; my mother Nusra, father Nabeel, brothers Ahmed, Mohamed and Babikir, and my lonely only sister Nidal.

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# Contents

<b>Abstract</b>	<b>vi</b>
<b>List of Figures</b>	<b>vii</b>
<b>1 Introduction</b>	<b>1</b>
<b>2 Introduction to General Theory of Relativity</b>	<b>3</b>
2.1 Why Do We Need Tensors? . . . . .	3
2.2 Important General Characteristic . . . . .	3
2.2.1 Tensor Rank . . . . .	3
2.2.2 Notation . . . . .	4
2.2.3 Cartesian Tensor . . . . .	4
2.2.4 Indicial Notation . . . . .	4
2.3 Rule Indices . . . . .	4
2.3.1 Index Rules . . . . .	5
2.3.2 Range Convention . . . . .	5
2.3.3 Summation Convention . . . . .	5
2.3.4 Free Index . . . . .	5
2.3.5 Dummy Index . . . . .	6
2.4 Important Symbols . . . . .	6
2.4.1 Kronecker Delta . . . . .	6
2.4.2 Levi Civita . . . . .	6
2.5 Covariant & Contravariant Vectors . . . . .	7

2.6	From Vectors To Tensors . . . . .	8
2.7	Tensor Definition . . . . .	8
2.7.1	Characteristics of Tensors . . . . .	9
2.7.2	Addition & Subtraction . . . . .	9
2.7.3	Outer Product . . . . .	9
2.7.4	Inner Product . . . . .	10
2.7.5	Symmetric & Anti-symmetric Tensors . . . . .	10
2.8	Why Do We Need Covariant Derivative? . . . . .	11
2.9	Covariant Derivative for Higher Rank Tensor . . . . .	13
2.10	The Metric Function And Tensor . . . . .	14
2.10.1	The Properties of The Metric Tensor . . . . .	15
2.11	Derivative of Basis Vector and Affine Connection . . . . .	15
2.12	The Relation Between $\Gamma$ & the Metric Tensor . . . . .	16
2.13	Calculating the Christoffel Symbols . . . . .	17
2.14	Intrinsic Dervative of A vector Along Curve . . . . .	19
2.15	Parallel Transport . . . . .	20
2.16	Geodesic . . . . .	21
2.17	The Curvature Tensor . . . . .	22
2.18	Properties of Rieman Tensor . . . . .	23
2.18.1	Ricci Tensor and Scalar . . . . .	24
2.18.2	Bianchi Identity . . . . .	24
2.19	Einstein's Field Equations (EFEs) . . . . .	24
2.19.1	The Equivalence Principle of General Relativity . . . . .	26

<b>3</b>	<b>Bianchi Models</b>	<b>28</b>
3.1	Introduction . . . . .	28
3.2	Bianchi Type-V Space-time Line Element . . . . .	28
3.3	Calculating the $\Gamma^s$ . . . . .	28
3.4	Calculating Ricci Tensor and Ricci Scalar . . . . .	29
3.4.1	The Einstein's Tensor . . . . .	35
3.5	The Energy Momentum Tensor . . . . .	35
3.5.1	The EFEs . . . . .	36
3.5.2	General Solution . . . . .	39
3.5.3	Solution Process . . . . .	41
3.5.4	Special Case . . . . .	44
<b>4</b>	<b>Conclusion</b>	<b>46</b>

# Abstract

The homogeneous anisotropic Bianchi type- $V$  cosmological model with variable gravitational and cosmological constants is investigated. Exact solutions of the Einstein field equations are presented in terms of adjustable parameter of quantum field in a curved and expanding background. We found that the Cosmological constant decreases as time increases whereas the gravitational constants increases respectively. The universe in this model approaches isotropy state at late period of time. A clear presentation for the physical and kinematical quantities of the model are also presented.