Quantum mechanical New Lasing Mechanisms

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Abstract

Quantum mechanical expressions for the amplification factor, deduced from the relation between the wave function and the dipole moment, are presented. It is shown that amplification of lasing process takes place when the electric dipole moment emitted photon exceeds the incident photon numbers, and when the lattice force exceeds the external force.

Introduction

Light amplification now plays an important role in our every day life. It has a wide variety of applications in industry, medicine and engineering. Therefore it is not surprising to find many papers dealing with amplification mechanisms in lasers production [1,2,3,4]. Most of the amplification processes that take place nowadays are based on the population inversion [5]. This makes a limitation on the materials used to induce laser. Thus there arises a need for new amplification mechanisms, so as to increase the possibility of discovering new lasing materials.

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To find new amplification mechanisms quantum mechanical equations are utilized. In section 2, the photon quantum equation in a resistive medium, within which atoms acts as electric dipole moments, is solved. The amplification factor is related to the electric polarization vector. In section 3 the amplification factor known to be related to the dipole moment. In sections 4 and 5 the amplification conditions for the quantum harmonic oscillator and hydrogen like atoms are discussed, Sections 5 and 6 are devoted to conclusions and discussions.

Amplification due to photon polarization

The photon wave function can be obtained by using the relativistic relation in the usual notation

$$\mathrm{E}^2 = p^2c^2 + m_0^2c^4 \tag{2.1}$$

But the photon is massless (m_0=0) and so

$$\mathrm{E} = pc \tag{2.2}$$

Energy and momentum operators $\hat{E}$ and $\hat{p}$ are obtained from the relation

$$\psi = Ae^{\frac{i}{\hbar}(px - Et)} \tag{2.3}$$

as

$$\hat{E} = i\hbar\frac{\partial}{\partial t}\,, \qquad \hat{p} = \frac{\hbar}{i}\nabla \tag{2.4}$$

Thus the free photon wave equation is given, according to (2.4) and (2.2), by

$$i\hbar\frac{\partial}{\partial t}\psi = \frac{\hbar c}{i}\nabla\psi \tag{2.5}$$

To check that this equation describes the free photon consider the solution with

$$p = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k \tag{2.6}$$

Where k stands for wave number. Substituting (2.3) in (2.5) and using (2.6) yields,

$$i\hbar\left(\frac{-i}{\hbar}\mathrm{E}\right)\psi = \frac{\hbar c}{i}\left(\frac{i}{\hbar}p\psi\right),\ \mathrm{E}\psi = pc\psi = \frac{hc}{\lambda}\psi = hf\psi$$

Where f is photon frequency
hence

$$\mathrm{E} = hf \tag{2.7}$$

which is the photon energy. The photon momentum can also be obtained by applying momentum operator to the wave function ψ $\frac{\hbar}{i}\nabla\psi = p\psi = \frac{h}{\lambda}\psi$

To see how the photon wave function looks like within a medium one can recall the Doppler and Compton effects which show that the photon frequency is affected by the motion of the radiation source beside the fields and collisions. Equation (2.4) can be made time-dependent by making the substitution

$$\psi = e^{-\frac{iE}{\hbar}t}u \tag{2.8}$$

One gets

$$Eu = \frac{\hbar c}{i}\nabla u \tag{2.9}$$

or

$$\nabla u = i\alpha u \tag{2.10}$$

where $\alpha = \frac{E}{\hbar c}$. The solution to eq (2.10) in one dimension takes the form

$$u = Ae^{\frac{iE}{\hbar c}x} \tag{2.10}$$

Or, since $E = \hbar\omega$, the photon wave function *u* becomes

$$u = Ae^{i\frac{\omega}{c}x} \tag{2.11}$$

The frequency of the oscillating polarized atoms is the same as the frequency of the photon, as suggested by the theory of the harmonic oscillator and proposed by classical electromagnetic theory. In this case the photon frequency can be found as in the classical harmonic oscillator theory, where polarization and external fields act on electrons, i.e

$$F = eC_0E - eE = e(C_O - 1)E_0e^{i\omega t} = \frac{e(C_0 - 1)}{x_0}E_0x_0e^{i\omega t} = \frac{e(C_0 - 1)}{x_0}E_0x = -kx$$

Where, F represents the force acting on the electron while C_0E stands for the polarization field by the momentum. *E* is the external field strength and C_0 is a proportion constants.

$$F = -m\omega^2 x \tag{2.12}$$

Thus, from $k = \frac{(1 - C_0)}{x_0}eE_0 = m\omega^2$, the photon frequency is given by

$$\omega = \pm\sqrt{\frac{1 - C_0}{x_o m}eE_0} \tag{2.13}$$

In case $C_0 > 1$

$$\omega = \pm\sqrt{\frac{(1 - C_0)}{x_o m}eE_0} = \pm i\omega_0 \,, \quad \omega_0 = \pm\sqrt{\frac{(C_0 - 1)}{x_o m}eE_0} \tag{2.14}$$

where *m* is the electron mass and ω is the photon frequency.

As a result equation (2.11)

$$u = A^2e^{\mp\frac{\omega_0}{c}x} \tag{2.15}$$

Since the intensity of radiation is, $I = hf\rho c = c|u|^2 = cA^2 e^{\mp\frac{2\omega_0}{c}x}$, one can take the plus sign for amplification to get

$$I = I_0 e^{\frac{2\omega_0}{c}x} = I_0 e^{\beta x} \tag{2.16}$$

Where $I_0 = cA^2$. In view of (2.14) lasing is possible where the amplification coefficient

$$\beta = \frac{2\omega_0}{c} = \frac{2}{c}\sqrt{\frac{C_0 - 1}{x_o m}E_0} \tag{2.17}$$

This requires the polarization field C_0E which results from polarized atoms to exceed the external field.

$$\omega_0 > 0 \quad , \quad P_0 = C_0 E_O > E_0 \tag{2.18}$$

This is not surprising so far as the polarization field reflects the number of photons emitted by the medium i.e

$$n_m = |C_0 E_O|^2 \tag{2.19}$$

This should exceed the number of incident photons,

$$n_i = |E_O|^2 \tag{2.20}$$

Whence

$|C_0 E_O|^2 > |E_0|^2$

or

$$n_m > n_i \tag{2.21}$$

Amplification and Dipole Moment

The amplification coefficient β can be defined in terms of the incident radiation intensity I_0 on a certain body beside with respect to transmitted intensity I according to the relation

$$I = I_0 e^{\beta z} \tag{3.1}$$

Where z stands for the body thickness. The amplification factor β can be related to Einstein coefficient B by utilizing the relation between radiation intensity I and β on one hand, and the relation between I_0 and β on the other hand. [5,6]

$$(N_2 - N_1)\rho BA\Delta z = \frac{\Delta IA}{hf}$$

With N_1,N_2 representing the number of atoms at the energy levels E_1, E_2 respectively. A is the area of the cross section, while ρ is the radiation density. Einstein coefficient B is the transition rate for one incident photon

$$(N_2 - N_1)\frac{\rho c}{c}B = \frac{1}{hf}\frac{\Delta I}{\Delta z} = \frac{1}{hf}\frac{dI}{dz} = (N_2 - N_1)\frac{I}{c}B = \frac{1}{hf}\frac{dI}{dz}$$, with

$$I = \rho c$$

$$(N_2 - N_1)\frac{Bhf}{c} = \frac{1}{I}\frac{dI}{dz} \tag{3.2}$$

This yields

$$I = I_0 e^{\beta z} \tag{3.3}$$

Where the amplification factor is defined to be

$$\beta = (N_2 - N_1)\frac{Bhf}{c} \tag{3.4}$$

Einstein coefficient in turn is related to the dipole moment μ where the rate of transition is given by [8]

$$B\rho = \frac{d|Cm|^2}{dt} = \frac{8\pi^3}{3h^2}\mu^2\rho \tag{3.5}$$

Thus

$$B = \frac{8\pi^3}{3h^2}\mu^2 \tag{3.6}$$

Where

$$\mu = \int \bar{u}_m \hat{H}_1 u_n dr \qquad (3.7)$$

with the u_m standing for the wave function and $\hat{H}_1$ for the perturbing Hamiltonian which is given by

$$\hat{H}_1 = \sum_j e_j x_j \qquad (3.8)$$

Where e_i and x_i represent the charge and displacement of the particle i .

Thus with the aid of (3.6) and (3.4) one gets

$$\beta = \frac{8\pi^3 f(N_2 - N_1)}{3hc} \mu^2 \qquad (3.9)$$

Harmonic Oscillator Gain Coefficients

According to classical electromagnetic theory oscillating dipoles emit electromagnetic radiation. The quantum mechanical treatment of harmonic oscillator also indicates that the energy of an oscillating body takes the form, [7]

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \qquad (4.1)$$

This means that the change in energy between successive levels is given by

$$\Delta E = \hbar\omega \qquad (4.2)$$

This means that if these emitted photons are in phase and coherent they may produce a laser beam. To see how this happens, one can utilize equations (3.9) and (3.7) by considering the wave function of the harmonic oscillator. As a result one gets [7]

$$\mu = \int \bar{u}_s x u_n dx = \frac{(n+1)}{2\alpha} \qquad , s = n+1 \tag{4.3}$$

Where n,s are quantum numbers

The above integral exists when studding the harmonic oscillator in the standard texts of quantum mechanics [7]

$$\alpha = \left(\frac{mk}{\hbar^2}\right)^{1/4} = \left(\frac{m\omega}{\hbar}\right)^{1/2} \tag{4.4}$$

Where

$$k = m\omega^2 \tag{4.5}$$

Hence the amplification coefficient is given by

$$\beta = \frac{8\pi^3 f(N_2 - N_1)}{3hc}\mu^2 = \frac{2\pi^3 f}{3hc}(N_2 - N_1)\left(\frac{\hbar^2}{mk}\right)^{1/2}(n+1)^2$$

$$= \frac{2\pi^2 f}{3}\frac{(N_2 - N_1)(n+1)^2}{cm\omega} = \pi\frac{(N_2 - N_1)(n+1)^2}{3m}$$

$$\beta = \frac{2\pi^3 f}{3h}\frac{(N_2 - N_1)(n+1)^2}{c}\left(\frac{\hbar}{mk}\right)(n+1)^2 \tag{4.6}$$

In view of the equation of motion

$$F = -kx = F_s + eE - F_r$$

$$F = -kx_0 e^{i\omega t} = (F_0 + eE_0 - \frac{m\nu_0}{\tau})e^{i\omega t} \tag{4.7}$$

where $F_s = F_0 e^{i\omega t}$ is the applied sound wave force, eE is the external electric force and $F_r = \frac{m\nu_0}{\tau}e^{i\omega t}$ is the resistive force, and v_0 is the max velocity. Thus k becomes

$$k = \frac{m\frac{\upsilon_0}{\tau} - eE_0 - F_0}{x} \tag{4.8}$$

In view of (4.7), (4.8) and (4.5) amplification is possible if

$$m\frac{\upsilon_0}{\tau} - eE_0 - F_0 > 0, \qquad F_r > F_e + F_s \tag{4.9}$$

The resistive force a gain dominates in complete agreement with classical electromagnetic theory. Another lasing condition can be obtained if one replaces the ordinary mass *m* in (4.7) by the effective mass m* in which the effect of crystal field is incorporated via the term F_l which represents the lattice force, which is assumed here appose the electric force F_e , by which the effective mass takes the form

$$m^* = \left(\frac{F_e}{F_e - F_l}\right)m \tag{4.10}$$

where F_e stands for the external force. This relation follows from the equations

$$ma = F_e - F_l \qquad m^* a = F_e \tag{4.11}$$

Replacing by *m** in (4.6) and substituting (4.10) one gets

$$\beta = \pi \frac{(N_2 - N_1)(n+1)^2}{m}\left(\frac{(F_e - F_l}{F_e}\right)e^2$$

when no inversion exists, the population in the exited state N_2 is less than that in ground state N_1, in this case

$$\beta = \pi \frac{(N_2 - N_1)(n+1)^2}{m}\left(\frac{(F_l - F_e}{F_e}\right)e^2 \qquad (4.12)$$

Thus by replacing m^* by m in (4.7) lasing takes place with no inversion when, $F_l > F_e$, i.e when the lattice force exceeds the external one.

Lasing from hydrogen-like Atoms

For atoms having very small nuclei or spherical nuclei their wave function can be described by the radial part. One can choose, for simplicity, the following functions [9]

$$R_{21} = \frac{1}{\sqrt{24}}\left(\frac{Z}{a}\right)^{\frac{5}{2}} re^{-\alpha r} \qquad R_{10} = 2\left(\frac{Z}{a_0}\right)^{3/2} re^{-\alpha r} \qquad (5.1)$$

With

$$\alpha = Z/2a \qquad (5.2)$$

The dipole moment takes the form

$$\mu = Ne\int R_{21} r R_{10} dr \quad = \frac{Ne}{\sqrt{6}}\left(\frac{z}{a_0}\right)^4 \int_0^\infty r^2 e^{-3\alpha r} dr$$

$$= \frac{Ne}{\sqrt{6}}\left(\frac{z}{a}\right)^4 \left[\left(\left[\frac{r}{-3\alpha}e^{-3\alpha r}\right]_0^\infty - \left[\frac{2}{9\alpha^2}e^{-3\alpha r}\right]_0^\infty - \frac{2}{27\alpha^3}\left[e^{-\alpha r}\right]_0^\infty\right)\right] \qquad (5.3)$$

$$\mu = \frac{-Ne}{\sqrt{6}}\left(\frac{z}{a}\right)^4 \frac{-2}{27\alpha^3} = \frac{2Ne}{27\sqrt{6}\alpha^3}\left(\frac{z}{a}\right)^4 \qquad (5.4)$$

where

$$a = \frac{\hbar^2}{\mu_0 e^2} \tag{5.5}$$

and μ_0 is the reduced mass defined by

$$\mu_0 = \frac{mM}{m+M} \tag{5.6}$$

where *m* and *M* are the electron and nuclei masses respectively. If *M* is proportional to *m*, $M=c_0m$ the reduced mass becomes

$$\mu_0 = \frac{c_0 m^2}{(c_0+1)m} = \frac{c_0}{(c_0+1)} m \approx m \tag{5.7}$$

When *M* is greater than *m* thus a and μ are given by

$$a = \frac{\hbar^2}{me^2}, \qquad \mu = \frac{2Ne}{27\sqrt{6}\alpha^3}\left(\frac{z}{a}\right)^4 \tag{5.8}$$

Thus according to equation (5.9) the gain coefficient is given by

$$\beta = \frac{8\pi^3 f}{3hc}(N_2 - N_1)\left[\frac{6}{27^2\alpha^6} N^2 e^2 \left(\frac{z}{a}\right)^8\right] \tag{5.9}$$

Equation (5.9) indicates that for hydrogen-like atoms, amplification depends on the atomic number z.

Discussion

For photons in dielectric medium equations (2.17), (2.18), (2.19), (2.20) and (2.21) show that amplification takes place if the electric polarization field exceeds the external field. This requires the induced photons to exceed the incident photons as shown by equation (2.21). The coherence condition is apparent, from the fact that the electric polarization field is in parallel.

Equation (3.9) relates the amplification factor to the dipole moment. For a quantum mechanical harmonic oscillator, amplification takes place if the resistive force dominates as indicated by equation (4.9). In this case collisions and hence excited atoms increase. These exited atoms emit a large number of photons which causes amplification. Equation (4.12) shows also that amplification can also be achieved, without inversion, if the crystal field exceeds the external field. This causes the induced electric field and hence the number of photons induced by the medium exceeds the external incident photons. Amplification for hydrogen like atoms requires population inversion as shown by equation (5.9)

Conclusions

The quantum mechanical treatment of the amplification factor shows new lasing mechanisms. It indicates that amplification may take place if the number of photon emitted by the polarization atoms in the medium exceeds the incident one. Lasing can also be achieved if the resistive crystal force exceeds all other forces. One can also amplify light if the crystal internal force exceeds the external one.

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